Equations : Chapter 2

① Format of Simple Equations = $\boxed{ax+b=0}$, $a\neq 0$.

② Format of Equations (2 unknown) $\boxed{\begin{array}{l} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{array}}$ $a\neq 0$.

For cross multiplication method, $\boxed{\dfrac{x}{b_1c_2-b_2c_1}=\dfrac{y}{c_1a_2-c_2a_1}=\dfrac{1}{a_1b_2-a_2b_1}}$

Solutions	1	∞/coincident	0
Conditions	$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

Format of Equations 3 unknow, $\boxed{\begin{array}{l} a_1x+b_1y+c_1z+d=0 \\ a_2x+b_2y+c_2z+d=0 \end{array}}$

③ Quadratic Equations, Format = $\boxed{ax^2+bx+c=0}$, $a\neq 0$

2 methods → Trial & error / Split middle term &

Formula method = $\boxed{\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}}$

Sum of Roots, $\alpha+\beta=-b/a$

Product of Roots, $\alpha\beta=c/a$.

Construct a Q.E
$\boxed{x^2-(\alpha+\beta)x+\alpha\beta=0}$

Discriminant of Q.E = $\boxed{b^2-4ac}$

Condition	Nature of Roots
$b^2-4ac=0$	Real and Equal
$b^2-4ac>0$	Real & Unequal
$b^2-4ac>0$ & a perfect sq.	Real, Unequal & Rational
$b^2-4ac>0$ & not a perfect sq.	Real, unequal & irrational.
$b^2-4ac<0$.	Imaginary & unreal

$a^3+b^3=(a+b)(a^2-ab+b^2)$
$=(a+b)[(a+b)^2-3ab]$

$a^3-b^3=(a-b)(a^2+ab+b^2)$
$=(a-b)[(a+b)^2-ab]$

$(a+b)^3=a^3+b^3+3ab(a+b)$

$(a-b)^3=a^3-b^3+3ab(a-b)$

$a^2+b^2=(a+b)^2-2ab$

when $(\alpha-\beta)$ is given in Qn use

$\boxed{(a+b)^2-(a-b)^2=4ab}$

Conjugate Pairs – $\boxed{\text{One root is } m+\sqrt{n} \text{ then other is } m-\sqrt{n}}$

(4) Cubic Equations, degree = 3

Format $= ax^3+bx^2+cx+d=0$, $a\neq 0$

Method → Trial & error.

Just glance → $\boxed{\alpha+\beta+\gamma = -b/a}$, $\boxed{\alpha\beta+\beta\gamma+\alpha\gamma = c/a}$, $\boxed{\alpha\beta\gamma = d/a}$

Chapter 1 – Ratio, Proportion, Indices & Logarithms

1) Ratio – E

Inverse – $a:b \rightarrow b:a$

Compounding (Multiply), $a:b$ & $c:d$ $= ac:bd$

Duplicate $= a:b \rightarrow a^2:b^2$

Triplicate $= a:b \rightarrow a^3:b^3$

Sub duplicate $= a:b \rightarrow \sqrt{a}:\sqrt{b}$

Sub triplicate $= a:b \rightarrow \sqrt[3]{a}:\sqrt[3]{b}$

Commensurable → $\sqrt{3}:\sqrt{2}$ In.

Continued Ratio → $a:b:c$

2) Proportion

$\underbrace{a:b=c:d}_{\text{Extremes}}$ or $a:\underbrace{b::c}_{\text{means}}:d$.

$ad = bc$ (Product of Extremes = Product of Means)

$a:b=b:c \Rightarrow b^2=ac$.

a → 1st Pro., b → mean proportional, c → 3rd Pro.

3) Properties of Proportion ($a:b=c:d$)

Invertendo → $b:a=d:c$

Alternando → $a:c=b:d$

Componendo → $\frac{a+b}{b}=\frac{c+d}{d}$

Dividendo → $\frac{a-b}{b}=\frac{c-d}{d}$

Componendo & Dividendo → $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

Addendo → $\frac{a+c+e+\dots}{b+d+f+\dots}=K$.

Subtrarendo → $\frac{a-c-e-\dots}{b-d-f-\dots}=K$

If $a:b=c:d=e:f=\dots=K$, then,

4) Indices

$a^m \times a^n = a^{m+n}$

$a^m/a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$(a\times b)^n = a^n \times b^n$

$a^0 = 1$

$\boxed{3^4=81}$

5) Log

$\log_a mn = \log_a m + \log_a n$

$\log_a \frac{m}{n} = \log_a m - \log n$

$\log_a m^n = n\log_a m$

$\log_b m = \frac{\log m}{\log b} = \frac{\log_a m}{\log_a b}$

$\boxed{\log_3 81 = 4.}$

$\log_b a \times \log_a b = 1.$

Solve all Q's from WB & IMP

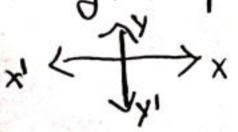
Chapter 3: Linear Inequalities

$x <$ left

$x >$ Right

$y <$ Down

$y >$ up.



Maths for finance

① $S.I = \frac{PTR}{100}$, $A = P + SI$ or $P\left[1 + \frac{Tr}{100}\right]$

② No. of Conversion Period per year.

Conversion Period.	Description or compounded.	No. of conversion periods in a year.
1 day	Daily	365
1 month.	Monthly	12
3 months.	Quarterly	4
6 months	Semi- annually.	2
12 months	Annually.	1

③ $C.I = P\left[(1+i)^n - 1\right]$, $i = \frac{r\,\%}{\text{no. of compounding period per year}}$ $A = P\left[1+\frac{i}{q}\right]^{n \times q}$

$A = P\left[(1+i)^n\right]$

$\boxed{\text{Calculator Trick} - \boxed{P} + \boxed{i\%} + \boxed{i\%} + \boxed{i\%} \ldots n \text{ times}}$

Effective Rate of Interest $= \left[(1+i)^n - 1\right]$

Depⁿ on WDV ⇒ ~~A[1 A = P[1+~~ $\boxed{A = P(1-i)^n}$

Diff. BTW CI & SI for $\boxed{\text{2 years}}$ ⟶ ~~Pr~~ Pr^2

④ $FV = CF(1+i)^n$

Annuity regular ⟶ end of each period
Annuity due ⟶ Beg. of each period

$FVAR = A_I \underbrace{\left[\frac{(1+i)^n - 1}{i}\right]}_{FVAF(n,i)}$

Use of FVAR ⟶ Final Value of Sinking fund or Savings amount to achieve target maturity value

For Accumulating certain sum

Annuity Regular = Annuity Immediate

$$FVAD = A_I \underbrace{\left[\frac{(1+i)^n - 1}{i}\right]}_{FVAF(n,i)}[1+i]$$

use → End of period maturity value.

If Qn silent → ANNUITY REGULAR

$$\boxed{PV = \frac{CF}{(1+i)^n}}$$

$$\boxed{PVAR = A_i \overbrace{\left[\frac{1}{i}\left[1-\frac{1}{(1+i)^n}\right]\right]}^{PVAF(n,i)}}$$

used → Calculation of loan amount, Leasing & Capital Exp (Appln)

$$\boxed{PVAF(n,i) = \boxed{1+i} \div = = = = = \ldots n\ times\ \boxed{GT}}$$

$$\boxed{PVAD = \left[A_I \times PVAF(n-1, i)\right] + A_I}$$

(5) Application of TVOM & other concepts

(a) Leasing – PV of annuity is compared with cash down price to decide if leasing is preferable.

(b) Capital Exp. Decision – PV of future benefits due to new assets are compared with purchase value of asset, to decide whether to purchase or not.

(c) Bond – PV of Interest income [PVAR] and maturity value [PV of Single CF] is compared with issue price of bond.

(d) $PVP = \frac{A_I}{i}$ or $\frac{A_I}{i/q}$.

(e) $PV\left(\text{Growing Perpetuity}\right) = \frac{A_i}{i - g}$ (i → discount rate, g → growth rate)

(f) NPV = PV of Cash Inflow – PV of cash outflow.
NPV ≥ 0, Accept Proposal.
NPV < 0, Reject Proposal.

(g) Real Rate of Return = Nominal Rate of Return – Inflation.

(h) CAGR → Annual growth as per CI

$$A = P(1+i)^n$$

Chapter - 5 : Permutations & Combinations

1. Multiplication (AND) - No. of ways of doing both things simultaneously / together = "$m \times n$" ways.
2. Addition (OR) - No. of ways of doing either of the jobs "$m+n$" ways
3. $n!$ or $\lfloor n = n(n-1)(n-2) \ldots . 3 \times 2 \times 1$ $\boxed{0! = 1}$
4. Special formula in factorial, $n! = n(n-1)!$ or $n(n-1)(n-2)!$
5. ${}^nP_r = \dfrac{n!}{(n-r)!}$, condition $\boxed{n \geq r}$
6. ${}^nP_n = n!$, Also $\boxed{{}^nP_n = {}^nP_{n-1}}$ → PYQ. $\boxed{{}^nP_r \cdot n = +ve \text{ Integer}}$
7. Special formula, $\boxed{(n+1)! - n! = n \times n!}$
8. Circular Permutations, $(n-1)!$
9. Circular Permutations when all objects are chosen out of n different objects such that no 2 persons have same 2 neighbou
   $(n-1)! \times \frac{1}{2}$ [SAME for necklace & garland etc]
10. when a particular object is not taken in Permutations,
    $\boxed{{}^{n-1}P_r}$
11. when a particular object is always included in arrangement,
    $\boxed{r \cdot {}^{n-1}P_{r-1}}$
12. ${}^{n-1}P_r$ (Always excluded) + $r \cdot {}^{n-1}P_{r-1}$ (Always included) = ${}^nP_r$
    ↓
    Total ways w/o restriction.
13. When never together = Total ways - Always Together
14. $p = \dfrac{n!}{n_1! n_2! n_3!}$

⑮ Summation of Possible No's (1,2,3,4)

(a) Find no. of numbers that can be formed (!) $= 4! = 24$

(b) Repeatition of each digit $= \dfrac{\text{Value of step a}}{\text{no. of diff. digits}} = \dfrac{24}{4} = 6.$

(c) Sum of digits $= 1+2+3+4 = 10.$

(d) Sum of digits × Repitition $= 10 \times 6 = 60.$

(e) Multiply (d) by 1111 or 111 for 4 digit or 3 digit no $= 60(1111) = 66660$

⑯ $^nC_r = \dfrac{n!}{(n-r)!\,r!} = \dfrac{^nP_r}{r!} \quad \therefore \boxed{r! = \dfrac{^nP_r}{^nC_r}}$

⑰ $^nC_0 = 1, \; ^nC_n = 1, \; ^nC_r = {}^nC_{n-r}.$

⑱ Special Formula $\boxed{^{n+1}C_r = {}^nC_r + {}^nC_{r-1}}$

⑲ Combinations of one or more out of n things (when 2 choices)

$\boxed{2^n - 1}$

⑳ Combination of zero or more out of n things.

$\boxed{2^n}$

㉑ Straight lines $= {}^nC_2$, with collinear $= {}^nC_2 - {}^mC_2 + 1$

㉒ Triangles $= {}^nC_3$, with collinear $= {}^nC_3 - {}^mC_3$

㉓ Parallelogram $= {}^nC_2 \times {}^mC_2$

㉔ Diagonals $= {}^nC_2 - n = \dfrac{n(n-3)}{2}$

㉖ Chords of Circle is $\boxed{^nC_2}$

㉕ Arrange → Includes all ~~Pr & Cr~~ ^nP_r & ^nC_r

Rearrange → Excludes the one in Qn from Total $^nP_r / {}^nC_r$.

㉖ No of factors $= (\text{power}+1) \times (\text{power}+1) \times (\text{power}+1)$

different factors = Exclude 1 from ↑

factors = Above formula

Chapter 6 : Sequence & Series

① nth element of sequence → use options for $n = 1, 2, 3, 4$ & verify the terms. [Eg of nth element $= 2n-1$]

② $\boxed{AP}$, format → $a, a+d, a+2d, a+3d, \ldots a+(n-1)d$.

$t_n = a+(n-1)d$.

3 terms a, b, c are in AP	$b-a = c-b$
	$a-b = b-c$
	$b = (a+c)/2$

calci Trick, $\boxed{a \pm d = = \ldots =}$
Term: 2nd 3rd nth

3 No's in AP, $\boxed{a-d, a, a+d}$

d can't be ZERO

$S_n = \frac{n}{2}\boxed{2a+(n-1)d} = \frac{n}{2}\left[a+t_n\right]$

Calci Trick for S_n $\boxed{a} \pm \boxed{d} = = = \ldots = \boxed{GT} + \boxed{a}$
Term 2nd 3rd 4th nth

First n natural no's, $S = \frac{n(n+1)}{2} = \Sigma n$.

First n odd no's, $S = n^2$

Square of first n natural no's, $S = \frac{n(n+1)(2n+1)}{6} = \Sigma n^2$

cubes of first n natural no's, $S = \left[\frac{n(n+1)}{2}\right]^2$

③ $\boxed{GP}$, format $= a, ar, ar^2, ar^3, \ldots, ar^{n-1}$.
1st 2nd 3rd 4th nth

$\boxed{r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}}}$ $\boxed{t_n = ar^{(n-1)}}$

calci trick → $r \times a = = = \ldots =$
Term: 2nd 3rd 4th nth.

3 No's in GP, $\boxed{\frac{a}{r}, a, ar}$

Calci trick for Sum

$r \times a = = = \ldots = \boxed{GT} + \boxed{a}$
2nd 3rd nth.

$S_n = \frac{a(1-r^n)}{(1-r)}$ or $\frac{a(r^n-1)}{(r-1)}$

$S_\infty = \dfrac{a}{1-r}$, Condition $\boxed{-1<r<1}$

3 terms x, y, z are in GP	$y/x = z/y$
	$x/y = y/z$
	$y^2 = xz$

(4) IMP POINTS

a) $AM = \dfrac{a+b}{2}$, $GM = \sqrt{ab}$, $HM = \dfrac{2ab}{a+b}$

$\boxed{AM > GM > HM}$ & $\boxed{GM = \sqrt{AM \cdot HM} \text{ or } (GM)^2 = AM \cdot HM}$

b) p^{th} term is q, q^{th} term is p, then r^{th} term in $\boxed{AP}$ is $\boxed{p+q-r}$

p^{th} term of $\boxed{GP}$ is x, q^{th} term is y, then n^{th} term is $\left[\dfrac{x^{(n-q)}}{y^{(n-p)}}\right]^{\frac{1}{p-q}}$.

c) when S_∞ is given $[7+77+777 \dots]$

→ find $S_2 = 7+77 = 84$

→ Use options, substitute $n=2$ & find option equal to 84.

d) when question gives, x, y & z are in AP assume $(1,2,3)$ & $x, y, (z+1)$ in GP assume $(1,2,4)$, then substitute values in options & see which condition is satisfied.

See W-B & IMP Qn

Chapter 14 : Central Tendency & Dispersion

A) change of origin = If a constant value is added or subtracted from each observations.

B) change of scale = If a constant value is multiplied or divided to all observations.

Measures of central Tendency :

① Arithametic Mean :

Discrete Observations, $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\Sigma x}{n}$

Simple FD (Discrete data), $\bar{x} = \frac{\Sigma fx}{N}$ $\boxed{x = \text{Individual Values}}$

Continuous FD, $\bar{x} = \frac{\Sigma fx}{N}$ $\boxed{x = \text{midpoint of C.I}}$

Assumed mean or Step deviation, $\boxed{\bar{x} = A + \frac{\Sigma fd}{N} \times C}$

where A = Assumed mean
Σfd = frequency deviation
$d = \frac{X - A}{c}$
C = class length.

Property – ① If observation constant (7, 7, 7), AM also constant (7)

② Algebric sum of deviations of a set of observation from their AM is zero, $\Sigma(x - \bar{x}) = 0$.

③ Affected both due to change of origin & change of scale

④ Combined AM, $\boxed{\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{(n_1 + n_2)}}$

Theory – ① Best measure of Central Tendency
② Based on all observations
③ Affected by sampling fluctuations
④ Amenable to mathematical property
⑤ Can't be used in case of open-end classification.
⑥ Rigidly defined.

• Pooled mean = Grouped mean.

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• Pooled mean = Grouped mean.

② Median : [Arrange in ascending order]

Discrete Observations, $n =$ odd, then middle term
$n =$ even, then average of 2 middle terms

Simple frequency dist, ① Arrange ascending order.
② Find LCF
③ Calculate $\left(\frac{N+1}{2}\right)^{th}$ observations.
④ Locate value in LCF & corresponding x value is median

Grouped FD, ① Find LCF
② Find N/2 & identify median class.
③ Calculate data for formula.
④ Apply formula

$$M_e = l_1 + \left[\frac{N/2 - N_1}{N_u - N_1}\right] C.$$

l_1	LCB of median class
N_u	Cum. frequency of median class
N_1	Cum. frequency of Pre-median class
C	class length

$(N_u - N_1) =$ Frequency of median class.

Properties - ① Sum of Absolute deviation is minimum, when the deviations are taken from median. $\boxed{\Sigma |X - Me| = min.}$
② Affected by both change of origin & scale.

Theory - ① Positional Average
② Not based on all observations
③ Not affected by sampling fluctuations.
④ Best measure of central tendency in case of open end classific.
⑤ Not affected by presence of extreme values.
⑥ Based on 50% of central values.

Partition Values

Name of PV.	No. of equal parts	No. of PV's	Symbol.
Median.	2	1	Me
Quartile	4	3	Q_1, Q_2, Q_3
Decile	10	9	$D_1, D_2, .. D_9$
Percentile.	100	99.	$P_1, P_2, P_3 ... P_{99}.$

Partition Values

Discrete observations, Rank calculation = $(n+1)p^{th}$ term.
Values of p depends on partition values.

#	Median	Quartile	Decile	Percentile
1st	1/2	1/4	1/10	1/100
2nd	—	2/4	2/10	2/100
Last	—	3/4	9/10	99/100.

Here 2.25th Term = 2^{nd} term + 0.25 (3^{rd} − 2^{nd} Term)

Grouped FD, (STEPS LIKE MEDIAN)

Q_1 class = $N/4$, $Q_1 = l_1 + \left[\frac{N/4 - N_1}{f}\right] c$. N_1 = LCF of Pre-median class

Q_3 class $= 3N/4$, $Q_3 = l_1 + \left[\frac{3N/4 - N_1}{f}\right] c$.

D_1 class $= N/10$, $D_1 = l_1 + \left[\frac{N/10 - N_1}{f}\right] c$.

D_9 class $= 9N/10$, $D_9 = l_1 + \left[\frac{9N/10 - N_1}{f}\right] c$.

P_1 class $= N/100$, $P_1 = l_1 + \left[\frac{N/100 - N_1}{f}\right] c$.

P_{99} class $= 99N/100$, $P_{99} = l_1 + \left[\frac{99N/100 - N_1}{f}\right] c$

③ **Mode**: Value occurs maximum times.
- 2 or more observations with same frequency → multiple modes (multimodal dist)
- 2 observations with same frequency → Bimodal dist
- If all obs. have same freq. → no mode.

Grouped FD, ① Modal Class = CI (class interval) corresponding to highest frequency.

② $M_O = l_1 + \left[\frac{f_0 - f_1}{2f_0 - f_{-1} - f_1}\right] c$

f_0 = f of modal class
f_{-1} = f of premodal class
f_1 = f of post modal class.

Property – ① All observation constant, then mode also constant.
② Mode is affected by both change of scale & origin.

Theory – ① Not based on all obs.
② Not rigidly defined or it is not uniquely defined
③ Not amenable to Mathematical Property.

Relationship between Mean, Median & Mode

① Symmetric Dist. = Mean = Median = Mode.

② Moderately Skewed Dist. (emperical method) = Mean − Mode = 3(Mean − Median)

Mode = 3Median − 2Mean.

④ Geometric Mean – n^{th} root of product of n positive observations.

Discrete obser, $G = (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{1/n}$

Frequency dist, $G = \left[x_1^{f_1} \cdot x_2^{f_2} \cdots x_n^{f_n}\right]^{1/N}$ $\boxed{N = \Sigma f}$

Property – ① $\log G = \frac{1}{n} \Sigma \log x$, Log of G of set of observations is the AM of log of observations.

② If all observations were constant, GM also constant.

③ If $z = xy$, then GM of z = GM of x · GM of y

④ If $z = x/y$, then GM of $z = \frac{\text{GM of } x}{\text{GM of } y}$

Theory – ① Not suited when '−ve' values are present (losses)

② Appropriate for rates having percentages (%)

③ Difficult to compute (to $\bar{x}$, Me & Mo)

⑤ Harmonic Mean – for a set of non zero observations, Reciprocal of AM of reciprocal of observations.

Discrete observations, $H = \frac{n}{\Sigma(1/x)}$ FQ $= \frac{N}{\Sigma(f/x)}$

Property – ① If all observations are constant, HM also constant.

② Combined HM, $H_c = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}} = \frac{n_1 + n_2 (H_1 H_2)}{n_1 H_2 + n_2 H_1}$

• Weighted HM $= \frac{\Sigma W}{\Sigma(W/x)}$

Theory – ① Not used when zero is in observations

② Appropriate for ratis other than Percentage (%)

* Note: HM & GM are used for calculating averages rates

Relationship between AM, GM & HM

Scenario.	Relation
All observations are same/identical	AM = GM = HM
Observations are distinct	AM > GM > HM
Question is silent	AM ≥ GM ≥ HM

Only 2 observation Then,

$AM \cdot HM = (GM)^2$

Weighted Averages

① Weighted AM = $\frac{\Sigma WX}{\Sigma W}$ ② Weighted GM = $(x_1^{W_1} \cdot x_2^{W_2} \cdot x_3^{W_3} \cdots x_n^{W_n})^{1/\Sigma W}$

③ Weighted HM = $\frac{\Sigma W}{\Sigma(W/X)}$

Measures of Dispersion (ONLY AFFECTED BY CHANGE OF SCALE) (CAN'T BE NEGATIVE)

① Range – Discrete = L − S , L = Largest obs, S = Smallest obs.
(Absolute measure) Grouped FD = L − S, L = UCB of last C.I
S = LCB of first C.I

Coefficient of Range = $\frac{L-S}{L+S} \times 100$. (Relative measure)

Properties – Not affected by change of origin
Affected by change of scale (only value, Ignore sign) −3x then x3
No Impact of sign of change of scale.
Can't be Negative : Note

Theory – Not based on all observation & ez to compute.

② Mean Deviation – Based on absolute deviation only.

Discrete obs, $MD_A = \frac{1}{n}\Sigma|X-A|$ A = appropriate central tendency

Freq. dist, $MD_A = \frac{1}{N}\Sigma f|X-A|$ (Absolute measure)

Coefficient of MD = $\frac{MD \text{ about } A}{A} \times 100$. (Relative measure)

Properties - ① It takes its minimum value when deviations are taken from median. (1st Property of Median)

② Change of origin - No affect, change of scale - affect of value not sign.

Theory - ① Based on all observations & absolute deviations.

② Improvement over range

③ Difficult to compute

④ Not amenable to Mathematical Property because of usage of modulus.

③ Standard deviation - Improvement over MD.

(Absolute)

Discrete obs., $\sigma_x = SD_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$

Freq. dist., $\sigma_x = SD_x = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{N}} = \sqrt{\frac{\Sigma f x^2}{N} - (\bar{x})^2}$

(Relative)

Coefficient of Variation $= \frac{SD_x}{\bar{x}} \times 100$ $\boxed{\text{Variance} = (SD)^2}$

SD of any 2 numbers $= \frac{\text{Range}}{2}$, SD for 1st n natural nos $= \sqrt{\frac{n^2-1}{12}}$

Property - ① If all obs. constant, then SD is ZERO

② No effect of change of origin but affected by change of scale in magnitude (ignore sign)

③ $SD_c = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$

$d_1 = \bar{x}_c - \bar{x}_1$, $d_2 = \bar{x}_c - \bar{x}_2$

Theory - Most useful measure of SD.

Measure of dispersion considered for finding a pooled measure of dispersion after combining several groups. {Combined SD}

If all obs. have same freq, then ignore freq.

Not used if data is open-ended.

④ Quartile Deviation

$QD_x = \frac{Q_3 - Q_1}{2}$ (Semi-Inter quartile range)

Interquartile range = $Q_3 - Q_1$

Median = $\frac{Q_3 + Q_1}{2}$

Coefficient of $QD_x = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$

Theory –
① Best measure of dispersion for open-end classification
② Less affected by sampling fluctuations
③ Not affected by presence of extreme observations
④ Based on only central 50% of observations.
⑤ QD not affected by change of origin, but only change of scale ignoring sign.

Relationship between ~~AM~~, SD, MD & QD

4SD = 5MD = 6QD or SD : MD : QD = 15 : 12 : 10

Note – ① change of scale & origin – central Tendency
change of Scale (× & ÷) – Dispersion (check 79th Qn)
② The more the coefficient of Variation less consistent the data will be.
③ If in Qn Var. is given of x & var of y is asked then first find SD of x ($\sqrt{\text{Var}x}$) & then find SD of y [change of Scale only] and then square SD of y to find Var. of y.

Chapter 7 – Sets, Relations & Functions

- $(1,99) \rightarrow$ Open set Interval (Except 1 & 99 all in Btw)

  $[1,99] \rightarrow$ Closed Interval set (All including 1 & 99)

  $\{1,99\} \rightarrow$ Ordinary Set (only 1 & 99)

- $a \in A$ (Belongs)

  $b \notin A$ (Not part of A)

  $A \subset B$ (A subset of B)

  $B \supset A$ (B superset of A)

  $A \subseteq B$ (Improper Subset)

★ No. of Possible subsets $\rightarrow 2^n$ (Power of Set) (no of relations)

No. of Proper Subsets $\rightarrow 2^n - 1$

- Null Set $\rightarrow \{\}$ or ϕ
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
- $n(A \cap B' \cap C') = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
- $(A \cap B)' = A' \cup B'$

  $(A \cup B)' = A' \cap B'$

- Complimentary $= P'$ or P^c

Reflexive (a,a), (b,b)	✓	✓	×	×	✓
Symmetric (a,b), (b,a)	×	✓	✓	✓	✓
Transitive (a,b), (b,c) (a,c)	×	×	×	✓	✓

Equivalence all 3.

R → Is equal to, Parallel to
S → Is parallel to, Reciprocal of
T → Is parallel to, Smaller than.

- Domain of $R^{-1} \rightarrow$ Range of R

  Range of $R^{-1} \rightarrow$ Domain of R

  (Domain → 1st Elements, Range → 2nd ele)

Mapping	1 to 1	1 to Many	Many to one
Function	✓	✗	✓

(First element can't be same but second can be same)

- Domain (Set of all 1st ele) → Inputs → Pre-img

  Codomain (Set of all 2nd ele) → Outputs → Image

Mapping		Range	
One-One	Many-1	Onto (Surjec)	Into (Inject)
diff ele diffing	2 or more ele have common images	Every ele in B has atleast one pre-img in A	If one element doesn't have a pre-img
		Range = Co.domain	Range ⊂ Codomain

- Bijective → Injective + Surjective
- Identity f → $f(x) = x$
- Constant f → $f(x) = 10$ eg. [x can be any value]
- Equal f → $f(x) = g(x)$
- Even f → $f(-x) = f(x)$

  Odd f → $f(-x) = -f(x)$

- Inverse f (Possible only if Bijective or one-one, onto)

  (a) $y = f(x)$ $[f(x) = 2x+1]$ $y = 2x+1$

  (b) find $x = \square$ $x = (y-1)/2$

  (c) Interchange x & y, $y = \square$ → Inverse of x. $\boxed{y = \frac{x-1}{2}}$

Chapter 7 - Sets, Relations & Functions

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Symmetric (a,b), (b,a)	×	✓	✓	✓	✓
Transitive (a,b), (b,c) (a,c)	×	×	×	✓	✓

Equivalence all 3.

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- Inverse f (Possible only if Bijective or one-one, onto)

  (a) $y = f(x)$

  (b) find $x = \square$

  (c) Interchange x & y, $y = \square$ → Inverse of x

  [$f(x) = 2x+1$]

  $y = 2x+1$

  $x = (y-1)/2$

  $\boxed{y = \frac{x-1}{2}}$

Chapter 15: Probability [READ THEORY & IMP from WB & PC]

① Classical Probability

- Subjective and Objective probability are the 2 broad divisions.
- The term 'chance' & probability are synonyms. – True
- Total no of possible outcomes $= p^q$ [p = no of events in 1 trial, q = no. of trials]
- $P(A) = \dfrac{\text{no of fav. outcomes}}{\text{Total no. of outcomes}}$, $P(A) = 1$ {Sure Event & $P(A) = 0$ {Impossible Event

  $\boxed{0 \le P(A) \le 1}$ $\boxed{P(\bar{A} \text{ or } A') = 1 - P(A)}$
- Playing Cards, Total 52 [13 in $\underbrace{\text{Spades, Clubs}}_{\text{Black}}$, $\underbrace{\text{Hearts, Diamond}}_{\text{Red}}$]

  KQJ → Face Cards.
- 2 Unbiased coins = HT, TT, HH, TH. (2^2)

  3 unbiased coins = HHH, HTH, HTT, THH, THT, TTH, TTT, HHT (2^3)
- odd in favour $= \dfrac{\text{favourable}}{\text{unfavourable}}$, odds against $= \dfrac{\text{unfavourable}}{\text{favourable}}$

② Set Based Probability

- $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{\text{no. of sample points in set A}}{\text{no. of sample points in Sample Space S}}$
- $\underbrace{P(A \cup B) = P(A) + P(B)}_{\text{2 Mutually Exclusive.}} - P(A \cap B)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$
- Expected frequency of Event A $= P(A) \times N$.
- $P(A - B)$ or $P(A \cap B') = P(A) - P(A \cap B)$
- $P(B - A)$ or $P(A' \cap B) = P(B) - P(A \cap B)$

③ Conditional Probability

$P(B/A) = \dfrac{P(A \cap B)}{P(A)}$ $(P(A) \ne 0)$ & $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ $(P(B) \ne 0)$

Compound Theorem for dependent events:

$P(A \cap B) = P(B/A)\, P(A)$

$P(A \cap B) = P(A/B)\, P(B)$

Compound Theorem for Independent events.

$P(A \cap B) = P(A) \cdot P(B)$

$P(A' \cap B) = P(A') \cdot P(B)$ or $P(B - A)$

$P(A \cap B') = P(A) \cdot P(B')$ or $P(A - B)$

$P(A' \cap B') = P(A') \cdot P(B')$ or $P(A \cup B)'$

[illegible]

Demorgan's law App. $\rightarrow P(A\cup B\cup C) = 1 - P(A\cap B\cap C)'$

$$= 1 - P(A'\cap B'\cap C')$$

$$= 1 - [P(A')\cdot P(B')\cdot P(C')]$$

Random Variable (RV)

- $\mu = E(X) = \sum PX, \quad \sum P = 1$
- $Var = \sigma^2 = E[X^2] - [E(X)]^2.$

$$\sigma^2 = \sum PX^2 - [\sum PX]^2.$$

If x is a RV & a, b are constant

a) $E(a) = a$

b) $E(ax) = aE(x)$

c) $E(ax+b) = aE(x) + b$

d) $Var(a) = 0$

e) $Var(ax) = a^2 Var(x)$

f) $Var(ax+b) = a^2 Var(x)$

$$\text{Relative frequency} = \frac{\text{Class frequency}}{\text{Total frequency}}$$

$$P(A) = \lim_{n\to\infty} \frac{f_A}{n}.$$

Chapter - 16 : Binomial Theoretical Distribution

① Binomial Distribution [Method of moments applied]

- Discrete & Bi-parametric with parameters n & p (2)
- pmf, $f(x) = P(X=x) = {}^nC_x \cdot p^x \cdot q^{n-x}$, $x = 0, 1, 2, \cdots, n$
  $q = 1-p$
- mean, $\mu = np$; variance, $\sigma^2 = npq$; SD, $\sigma = \sqrt{npq}$
- Mean is ALWAYS MORE than Variance. [$\mu > \sigma^2$]
- Maximum Value of Variance = $n/4$.
- Symmetrical if $p = 1/2$
- Mode $= (n+1)p \rightarrow$ If result is $\longrightarrow$ Non Integer (value in decimals) $\longrightarrow$ one mode consider only the number & not points

[UNI-modal or Bimodal]

Integer (~~whole &~~ +ve no) $\rightarrow$ 2 MODES $\rightarrow$ $\boxed{(n+1)p \text{ \& } [(n+1)p]-1}$

- Additive identity, If $X \sim B(n_1, p)$ & $Y \sim B(n_2, p)$ then $X+Y \sim B(n_1+n_2, p)$
- Tends to
  - $\rightarrow$ Poisson $\rightarrow n = \infty, p = 0, np = \lambda$
  - $\rightarrow$ Normal $\rightarrow n = \infty$, p almost equal to q.

② Poisson Distribution [Unimodal or Bimodal]

- Discrete & uniparametric distribution.
- Always positively skewed.
- m - given or np.
- pmf, $P(X=x) = \dfrac{e^{-m} \cdot m^x}{x!}$, X can take whole no. value from 0 to n (∞)
- $\mu = m$, $\sigma^2 = m$, SD/$\sigma = \sqrt{m}$.

$\boxed{e = \text{euler's constant} = 2.71828}$

- Mode $\longrightarrow$ Integer $\longrightarrow \boxed{m \text{ \& } m-1}$
  $\longrightarrow$ Non integer $\longrightarrow \boxed{m}$

$\boxed{m \text{ can't be NEGATIVE}}$

- Additive Ppty., If $X \sim P(m_1)$ & $Y \sim P(m_2)$ then $X+Y \sim P(m_1+m_2)$
- If m is large, poisson dist $\longrightarrow$ Normal dist.

Chapter - 16 : ~~Binomial~~ Theoretical Distribution

① Binomial Distribution [Method of moments applied]

- Discrete & Bi-parametric with parameters n & p (2)
- pmf, $p(x) = P(X=x) = {}^nC_x \cdot p^x \cdot q^{n-x}$, $x = 0, 1, 2, \ldots, n$
  $q = 1-p$
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- Mean is ALWAYS MORE than Variance. $[\mu > \sigma^2]$
- Maximum Value of Variance $= n/4$.
- Symmetrical if $p = 1/2$
- Mode $= (n+1)p$ → If result is → Non Integer (value in decimals) → one mode consider only the number & not points

[UNI-modal or Bimodal]

If result is → Integer (~~whole &~~ +ve no) → 2 MODES → $\boxed{(n+1)p \ \& \ [(n+1)p] - 1}$

- Additive identity, If $X \sim B(n_1, p)$ & $Y \sim B(n_2, p)$ then $X+Y \sim B(n_1+n_2, p)$
- Tends to → Poisson → $n = \infty$, $p = 0$, $np = \lambda$
  Tends to → Normal → $n = \infty$, p almost equal to q.

② Poisson Distribution [Unimodal or Bimodal]

- Discrete & uniparametric distribution.
- Always positively skewed.
- m - given or np.
- pmf, $P(X=x) = \dfrac{e^{-m} \cdot m^x}{x!}$, X can take whole no. value from 0 to $n(\infty)$

$\boxed{e = \text{eulers' constant} = 2.71828}$

- $\mu = m$, $\sigma^2 = m$, SD/$\sigma = \sqrt{m}$.
- Mode → Integer → $\boxed{m \ \& \ m-1}$
  Mode → Non integer → $\boxed{m}$

$\boxed{m \text{ can't be NEGATIVE}}$

- Additive Ppty., If $X \sim P(m_1)$ & $Y \sim P(m_2)$ then $X+Y \sim P(m_1+m_2)$
- * m is large, poisson dist → Normal dist. *

③ Normal Distribution (AKA GAUSSIAN)

- When RV is continous & Bi-parametric. (μ & σ^2)
- Mean = Median = Mode = μ (Symmetrical dist.)
- Mean deviation $= 0.8\sigma$, QD $= 0.675\sigma$
- Quartiles $\rightarrow Q_1 = \mu - 0.675\sigma$ & $Q_3 = \mu + 0.675\sigma$
- Ration BTW SD : MD : QD = 15 : 12 : 10.
- PDF, $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2 \times \frac{1}{2}}$
- Normal curve is also called as probability curve. & is Bell shape.
- Points of Inflexions are $\mu - \sigma$ & $\mu + \sigma$
- Area under normal curve is 1 / unity

From	To	Avg Prob
μ	$\mu+\sigma$	34.135%
$\mu+\sigma$	$\mu+2\sigma$	13.59%
$\mu+2\sigma$	$\mu+3\sigma$	2.14%
$\mu+3\sigma$	∞	0.135%

From	To	Avg Prob
$\mu-\sigma$	$\mu+\sigma$	68.27%
$\mu-2\sigma$	$\mu+2\sigma$	95.45%
$\mu-3\sigma$	$\mu+3\sigma$	99.73%

- Additive property of Normal Distribution:

  $X \sim N(\mu_1, \sigma_1^2)$ & $Y \sim N(\mu_2, \sigma_2^2)$ then $X+Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

④ Standard Normal Distribution

- Condition, $\mu = 0$ & $\sigma = 1$
- $\boxed{\text{Mean} = \text{Median} = \text{Mode} = 0}$
- Convert X to Z, $Z = \dfrac{X-\mu}{\sigma}$
- $\sigma^2 = 1$ as $\sigma = 1$
- MD = 0.8 (as $\sigma = 1$)
  QD = 0.675
- Points of inflexion are -1 & $+1$
- Cumulative distribution function, $F(x) = P(X \leq x)$ or $\varphi(K) = P(X \leq K)$

  $F(x)$ or $\varphi(K)$ gives area from $-\infty$ to point K in a Std. dist.

Chapter - 17 : Correlation & Regression

Theory - ① Bivariate data - Data collected from/on 2 variable simultaneously & corresponding frequency derived from it is Bivariate FD.

② Marginal Distribution - Frequency dist. of one variable (X or Y) accross the other variables full range of values

No. of Marginal Dist. = 2

③ Conditional Dist. - FD of one variable (X or Y) accross a particular sub population of the other variable.

No. of conditional dist. = m+n

No. of Cells = m×n

④ Correlation analysis aims at establishing relation between 2 variables & measuring the extent of relation between 2 variables

⑤ Spurious correlation is correlation between 2 variables having no causal relation.

⑥ Scatter diagram useful for linear & non-linear correlation (Curvilinear) also. It helps find nature & relative strength of correlation.

⑦

Karl Pearson's Coefficient of Correlation

- used to understand nature as well as accurate strength/amount of correlation
- $r = 1$, Perfect Positive & $r = -1$, Perfect -ve
- $r_{xy} = \frac{Cov(x,y)}{(\sigma_x \cdot \sigma_y)}$, $Cov.(x,y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} = \frac{\Sigma xy}{n} - \bar{x}\cdot\bar{y}$

Property - ① unit free measure

② Value lies from -1 to $+1$ (Inclusive)

③ No impact of change of origin.

④ No impact of value of change of scale, but if change of scale of both variables are of different sign. then sign of r will also change

$x \xrightarrow[\text{Scale}]{\text{chng of}} u$ $+$ $-$ $-$ $+$

$y \xrightarrow[\text{scale}]{\text{chng of}} v$ $-$ $+$ $-$ $+$

$\underbrace{\quad}_{r \text{ will change sign}}$ $\underbrace{\quad}_{r \text{ won't change sign \& remains same}}$

* Product of SD of X & Y should be ~~less~~ MORE THAN OR EQUAL to Covariance.

* Used to find Correlation for linear only

③ Normal Distribution (AKA GAUSSIAN)
(parametric) (μ & σ^2)

- $y = a + bx$, $r_{xy} = +1$ if $b > 0$
- $r_{xy} = -1$ if $b < 0$.
- (R_{xy} = Perfect +/−) (a need not be positive)
- Covariance can be either positive, negative or zero.

Spearman's Rank Correlation Coefficient

- Correlation between 2 attributes (Beauty) ~~marks~~

① $r_R = 1 - \dfrac{6 \sum d^2}{n(n^2-1)}$ d = difference of ~~marks~~ ranks

② $r_R = 1 - \dfrac{6(\sum d^2 + A)}{n(n^2-1)}$, $A = \dfrac{\sum (t^3 - t)}{12}$ A = adjustment value, t = tie length.
(In case of tie)

Coefficient of Concurrent Deviations

- Very quick, simple & casual method of finding correlation when not serious about magnitude of 2 variables

$$r_c = \pm\sqrt{\pm\left(\frac{2c - m}{m}\right)}$$

c = no. of concurrent deviation (Same direction)
m = no. of pairs compared, ie $(n-1)$

limit → BTW −1 & +1, inclusive.

Regression Basics

- Regression analyses is concerned with establishing a mathematical relationship between 2 variables & predicting value of dependent variable for given value of independent variable.
- 2 Regression lines when identical $r = -1$ or $+1$.

Formula of Regression lines | **Regression Equations = 2**

Estimation of Y when X is given

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

b_{yx} = Regression coefficient of Y on X

$$b_{yx} = r \cdot \frac{SD_y}{SD_x} = \frac{Cov(x,y)}{(SD_x)^2}$$

Estimation of X when Y is given.

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

b_{xy} = Regression coefficient of X on Y.

$$b_{xy} = r \cdot \frac{SD_x}{SD_y} = \frac{Cov(x,y)}{(SD_y)^2}$$

$y = a + bx$, $r_{xy} = +1$ if $b > 0$ • Covariance can be either

Properties – ① Change of ~~Scale~~ origin – No impact, chang of scale impact of both magnitude & sign.

$$b_{uv} = b_{xy} \times \frac{\text{change of scale of } x}{\text{change of scale of } y} \qquad b_{vu} = b_{yx} \times \frac{\cos y}{\cos x}$$

② Two regression lines (if not identical) will intersect at the point (means)

③ Correlation coefficient is GM of regression coefficients,

$$r_{xy} = \pm\sqrt{b_{xy} \cdot b_{yx}}$$

r_{xy}, b_{xy}, b_{yx} all have same sign

- Product of regression coefficients b_{xy} & b_{yx} should be less than one or unity.
- Methods to form regression equations is called as method of least squares.

Y on X is formed by minimisation of vertical distances } Scatter
X on Y is formed by minimisation of horizontal distances } Diagram

Difference between estimated & observed value is error/residue.

Error / Residue can be +ve, zero or −ve.

Coefficient of Determination & Non Determination

- Coefficient of Determination / Accounted Variance / Explained Variance $= (r_{xy})^2$
- Coefficient of non determination / unaccounted variance / unexplained variance $= 1 - (r_{xy})^2$

- Spearman's Rank Correlation – Incorrect $\Sigma d^2 - (\text{wrong value})^2 + (\text{correct value})^2 =$ correct Σd^2

- Correlation Concurrent deviations

$$\text{Standard Error} = \frac{1 - r^2}{\sqrt{N}}, \quad \text{Probable Error} = 0.675 \times \frac{1 - r^2}{\sqrt{N}}$$

Coefficient of determination is (BOTH BELOW)

$$r^2 = 1 - \frac{\text{unexplained variance}}{\text{Total Variance}}, \quad r^2 = \frac{\text{Explained Variance}}{\text{Total Variance}}$$

Chapter - 18 : Index Numbers [READ W.B FOR THEORY & MCQ]

1. Index Numbers are a series of numerical figures which show the relative position.
2. Fisher's Index no. is an Ideal index no. c/z it satisfies TRT & FRT
3. Test of adequacy/consistency (4) – unit, TRT, FRT, Circular Test
4. P_{01} = 1 on 0 , P_{10} = 0 on 1
5. Circular test is satisfied by simple GM of price relatives & weighted aggregative with fixed weights.
6. Fisher's ideal index doesn't satisfy Circular test
7. Circular test is an extension of TRT [$P_{01} \times P_{12} \times P_{20} = 1$]
8. Index no's are expressed as percentages.
9. CLI is always weighted index no.
10. Index no's are helpful in framing economic policies, revealing trend, forecasting, comparison.
11. Circular test is known for shift base index no.
12. Laspeyre's index no. is based on last yr (base) weights.
13. Purchasing power of money is reciprocal of price index no.
14. Base Period is considered as POINT OF REFERENCE.
15. Weighted GM of relative satisfies TRT
16. Unit test is not satisfied by Simple Aggregative.
17. Best Avg for construction of Index no. is GM. (Most used – AM)
18. Fisher's method of construction of Index no's uses GM
19. when product on Price index & Qty index is equal ~~to~~ value index then the test hold is FRT.
20. Two opposite indices are reciprocal of each other in case of TRT ($P_{01} = 1/P_{10}$)
21. Splicing means constructing one continous series from two different indices on the basis of common sense.

FORMULA'S

① Price Relative $= \frac{P_n}{P_0} \times 100$.

③ Chain Relative/Indices $= \frac{P_1}{P_0}, \frac{P_2}{P_0}, \ldots, \frac{P_n}{P_0}$ (Year/Base)

② Link Relative $= \frac{P_1}{P_0}, \frac{P_2}{P_1}, \ldots, \frac{P_n}{P_{n-1}}$
(Next/Previous)

④ Chain Index/Relatives $= \frac{\text{Link Relative of CY} \times \text{chain Index of PY}}{100}$.
$\left[\frac{P_2}{P_1} \times \frac{P_1}{P_0} = P_2/P_0\right]$

⑤ Shifted Price Index $= \frac{\text{Original Price Index}}{\text{Price index of year on which it has to be shifted}} \times 100$.

⑥ Simple Aggregative Method $= \frac{\Sigma P_n}{\Sigma P_0} \times 100$.

⑦ Simple Avg of Relatives method $= \frac{\Sigma(P_n/P_0)}{N}$ $\boxed{N/n = \text{No of Years}}$

⑧

Method's name	Remark	Formula
Laspeyre's Index	Weight - Base Yr Qty	$\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$
Paasche's Index	Weight - CY Qty.	$\frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n} \times 100$
Marshall Edgeworth's Index.	Weight - Sum of BY Qty. & CY Qty.	$\frac{\Sigma P_n (Q_0 + Q_n)}{\Sigma P_0 (Q_0 + Q_n)}$ 100
Fisher's Index	GM of ~~Price~~ Laspeyres' & Paasche's Index.	$\sqrt{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}} \times 100$.
Bowley's Index	AM of Laspeyre's & Paasche's Index	$\frac{\frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} + \frac{\Sigma P_n Q_n}{\Sigma P_0 Q_n}}{2}$ $\frac{L+P}{2}$

⑨ Weighted Aggregative of Price Relatives $= \frac{\Sigma \frac{P_n}{P_0} \times P_0 Q_0}{\Sigma P_0 Q_0} \times 100 = \frac{\Sigma P_n Q_0}{\Sigma P_0 Q_0} \times 100$
(Same as Laspeyres')

⑩ Deflated Value $= \frac{\text{Current Value}}{\text{Price Index of CY}}$